Assume a representative firm combines capital and labor to produce output with Cobb-Douglas technology:

$$y_t = f(k_t, h_t) = k_t^{\theta} h_t^{1-\theta}$$

where k_t is physical capital and h_t is labor hours. The firm's problem is to choose capital and labor demand to maximize profits:

$$\max_{k_t, h_t} f(k_t, h_t) - rk_t - wh_t$$

where r is the capital rental rate and w is the wage. The capital is owned by households and rented to the firm. The firm's first-order conditions set the wage and the rental rate equal to the marginal products of labor and capital, respectively.

Assume a measure 1 of identical households with log utility:

$$u(c_t, h_t) = \ln(c_t) + B\ln(1 - h_t)$$

where c_t is period consumption. The parameter B governs the utility of leisure.

Households choose consumption and labor supply to maximize lifetime utility:

$$\max_{c_t, h_t} \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

subject to the period budget constraint

$$c_t + k_{t+1} - k_t \leq wh_t + rk_t$$

The budget constraint simply says that consumption plus investment must be less than earnings from labor and capital rental.

Observe that the notation above assumes market clearing. The firm's and households' first-order conditions for capital imply that steady state labor is given by

$$\bar{h} = \left[\frac{1}{\beta\theta} - \frac{1}{\theta}\right]^{\frac{1}{1-\theta}} \bar{k}$$

The aggregate resource constraint and the firm's and households' first-order conditions for labor imply that steady state consumption is given by

$$\bar{c} = \left[\frac{1}{\beta\theta} - \frac{1}{\theta}\right] \bar{k}$$

The aggregate resource constraint and the firm's and households' first-order conditions for labor also imply the following:

$$\frac{B}{1 - \bar{h}} = \frac{1 - \theta}{\bar{h}}$$

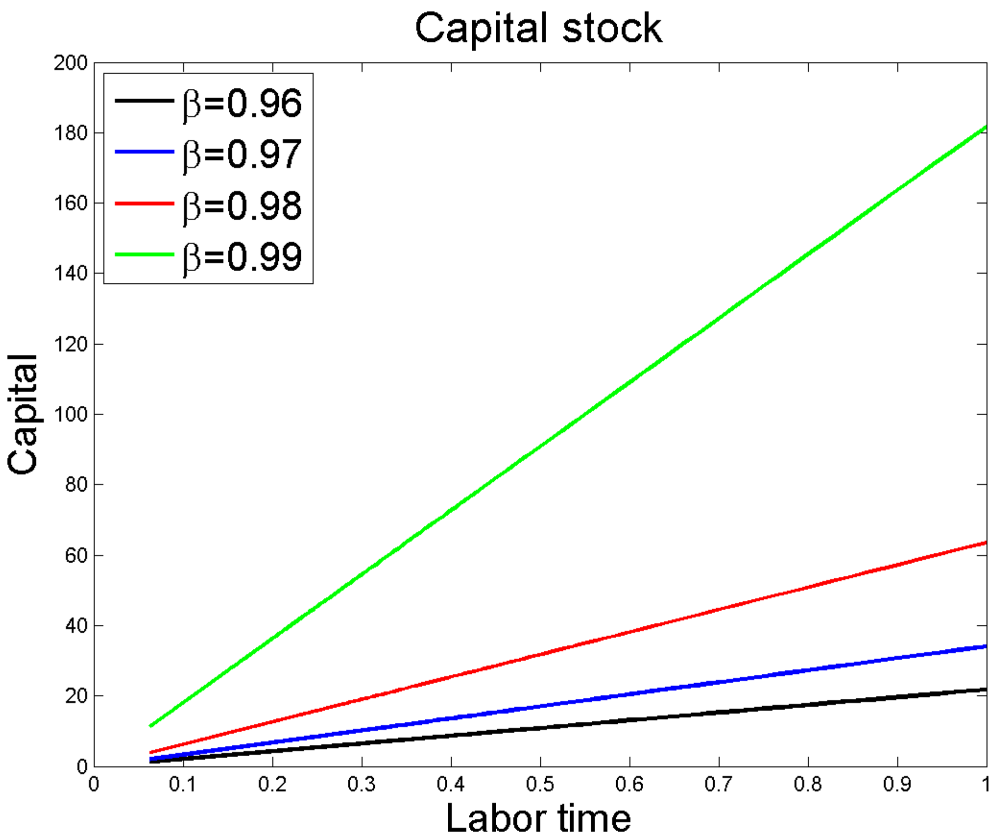
Combining this with steady state labor and consumption yields steady state capital:

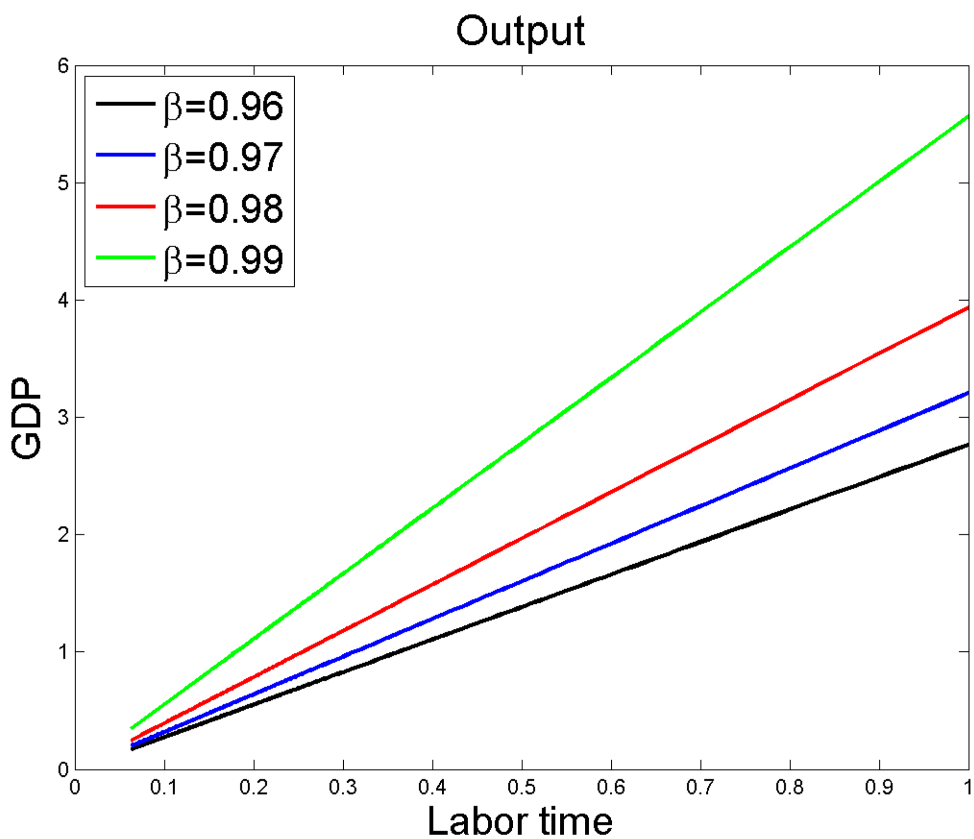
$$\bar{k} = \frac{(1-\theta)}{G(B+1-\theta)}$$

where

$$G = \left[\frac{1}{\beta\theta} - \frac{1}{\theta}\right]^{\frac{1}{1-\theta}}$$

Now recall that steady state output is given by $\bar{y} = \bar{k}^{\theta}\bar{h}^{1-\theta}$. Calibrate $\theta = 0.33$. Solve the steady state for values of B ranging from 0 to 10 and for reported values of β to generate the chart in the post.





Output
β=0.96
β=0.97
β=0.98
β=0.99
GDP
Labor time
0
1
2
3
4
5
6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9